GCE AS/A LEVEL

2305U30-1

S23-2305U30-1

FRIDAY, 26 MAY 2023 – AFTERNOON

FURTHER MATHEMATICS – AS unit 3
FURTHER MECHANICS A

1 hour 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9·8 ms^{-2}.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 70.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

Additional Formulae for 2023

Laws of Logarithms

$\log_a x + \log_a y \equiv \log_a(xy)$

$\log_a x - \log_a y \equiv \log_a\left(\frac{x}{y}\right)$

$k\log_a x \equiv \log_a\left(x^k\right)$

Sequences

General term of an arithmetic progression:

$$u_n = a + (n-1)d$$

General term of a geometric progression:

$$u_n = ar^{n-1}$$

Mensuration

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :

$$s = r\theta \qquad A = \frac{1}{2}r^2\theta$$

Calculus and Differential Equations

Differentiation

Function	Derivative
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$

Integration

Function	Integral
$f'\left(g(x)\right)g'(x)$	$f\left(g(x)\right) + c$

Area under a curve $= \int_a^b y\,\mathrm{d}x$

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. One end of a light elastic string, of natural length 0·2 m and modulus of elasticity $5g$ N, is attached to a fixed point *O*. The other end is attached to a particle of mass 4 kg. The particle hangs in equilibrium vertically below *O*.

(a) Show that the extension of the string is 0·16 m. [2]

(b) The particle is pulled down vertically and held at rest so that the extension of the string is 0·28 m. The particle is then released. Determine the speed of the particle as it passes through the equilibrium position. [8]

2. At time $t = 0$ seconds, a particle *A* has position vector $(6\mathbf{i} + 21\mathbf{j} - 8\mathbf{k})$ m relative to a fixed origin *O* and is moving with constant velocity $(3\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ ms^{-1}.

(a) Write down the position vector of particle *A* at time t seconds and hence find the distance *OA* when $t = 5$. [4]

(b) The position vector, $\mathbf{r}_B$ metres, of another particle *B* at time t seconds is given by

$$\mathbf{r}_B = 3\sin\left(\frac{t}{2}\right)\mathbf{i} - 3\cos\left(\frac{t}{2}\right)\mathbf{j} + 5\mathbf{k}.$$

(i) Show that *B* is moving with constant speed.

(ii) Determine the smallest value of t such that particles *A* and *B* are moving perpendicular to each other. [7]

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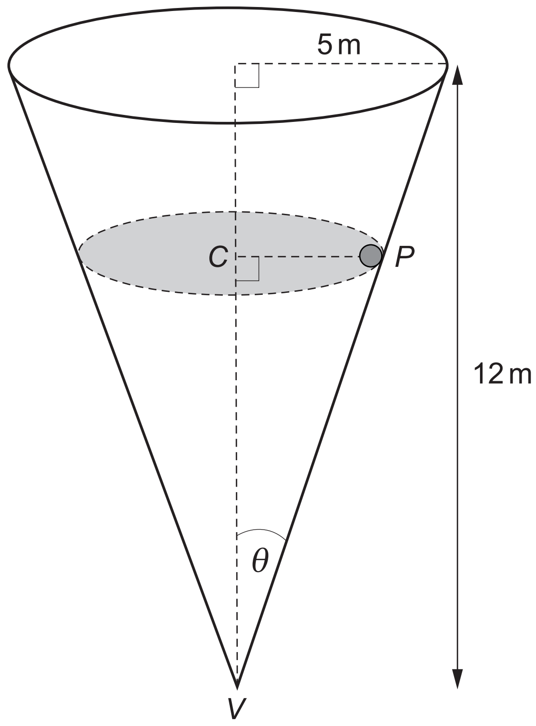
TURN OVER

3. The diagram below shows a hollow cone, of base radius 5 m and height 12 m, which is fixed with its axis vertical and vertex V downwards. A particle P, of mass M kg, moves in a horizontal circle with centre C on the smooth inner surface of the cone with constant speed $v = 3\sqrt{g}$ ms^{-1}.

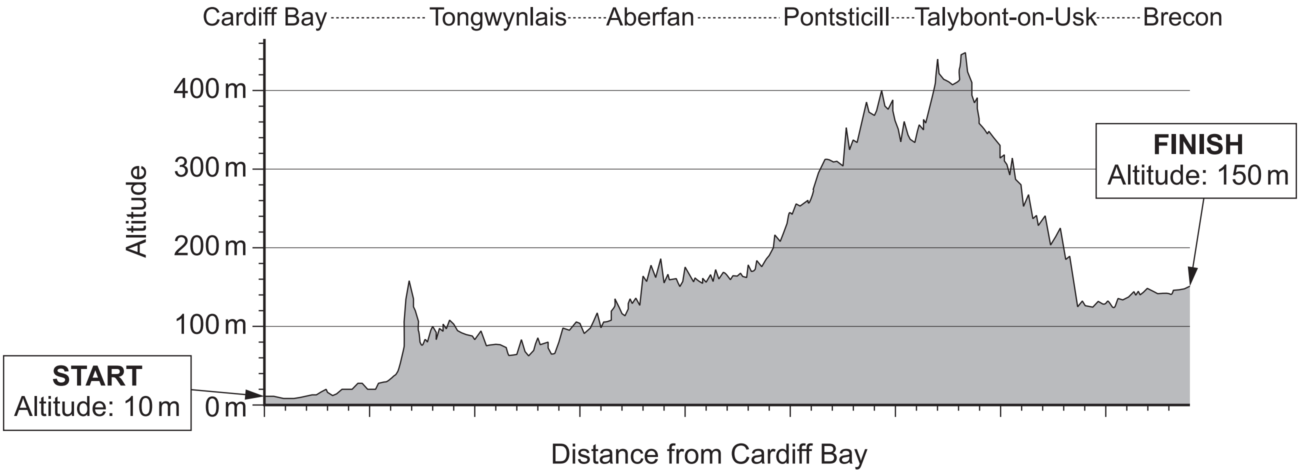


(a) Show that the normal reaction of the surface of the cone on the particle is $\frac{13Mg}{5}$ N. [4]

(b) Calculate the length of CP and hence determine the height of C above V. [6]

4. Geraint is a cyclist competing in a race along the Taff Trail. The Taff Trail is a track that runs from Cardiff Bay to Brecon. The chart below shows the altitude (height above sea level) along the route.

Altitude Chart of the Taff Trail



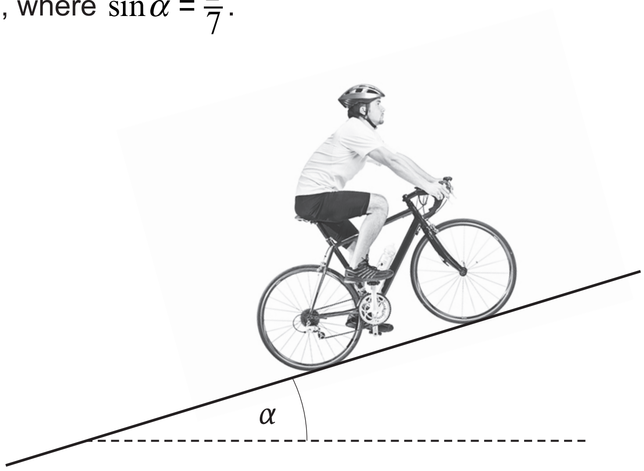
Geraint starts from rest at Cardiff Bay and has a speed of $10\,\mathrm{ms}^{-1}$ when he crosses the finish line in Brecon. Geraint and his bike have a total mass of 80 kg. The resistance to motion may be modelled by a constant force of magnitude 16 N.

(a) Given that 1440 kJ of energy is used in overcoming resistances during the race,

(i) find the length of the track,

(ii) calculate the work done by Geraint. [8]

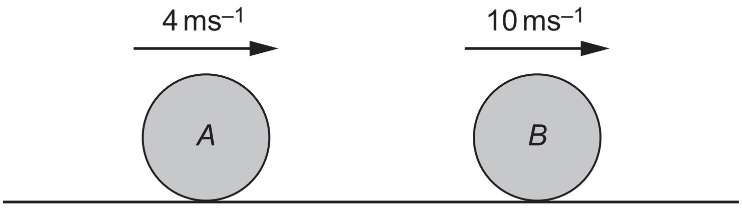
(b) The steepest section of the track may be modelled as a slope inclined at an angle α to the horizontal, where $\sin\alpha = \frac{2}{7}$.



Geraint is capable of producing a maximum power of 250 W. Find the maximum speed that Geraint can attain whilst travelling on this section of the track. [5]

TURN OVER

5. The diagram below shows two spheres *A* and *B*, of equal radii, moving in the same direction on a smooth horizontal surface. Sphere *A*, of mass 3 kg, is moving with speed 4 ms^{-1} and sphere *B*, of mass 2 kg, is moving with speed 10 ms^{-1}.



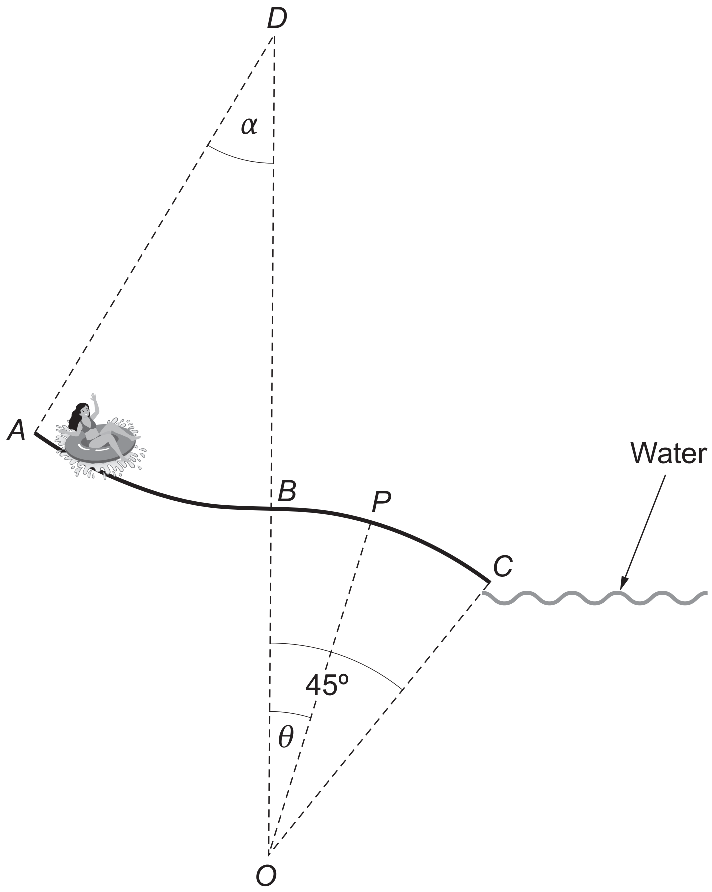
Sphere *B* is then given an impulse after which it moves in the opposite direction with speed 6 ms^{-1}.

(a) Calculate the magnitude and direction of the impulse exerted on *B*. [3]

Sphere *B* continues to move with speed 6 ms^{-1} so that it collides directly with sphere *A*. The kinetic energy lost due to the collision is 45 J.

(b) Calculate the speed of *A* and the speed of *B* immediately after the two spheres collide. State the direction in which each sphere is moving relative to its motion immediately before the collision. [8]

6. The diagram shows a slide, *ABC*, at a water park. The shape of the slide may be modelled by two circular arcs, *AB* and *BC*, in the same vertical plane. Arc *AB* has radius 7 m and subtends an angle α at its centre *D*, where $\cos\alpha = \frac{9}{10}$. Arc *BC* has radius 5 m and subtends an angle of 45° at its centre, *O*. The straight line *DBO* is vertical.



Users of the slide are required to sit in a rubber ring and are released from rest at point *A*. A girl decides to use the slide. The combined mass of the girl and the rubber ring is 50 kg.

(a) When the rubber ring is at a point *P* on the circular arc *BC*, its speed is $v\,\mathrm{ms}^{-1}$ and *OP* makes an angle θ with the upward vertical.

(i) Show that $v^2 = 111 \cdot 72 - 98\cos\theta$. [4]

(ii) Find, in terms of θ, the reaction between the rubber ring and the slide at *P*. [4]

(iii) Show that, according to this model, the rubber ring loses contact with the slide before reaching *C*. [3]

(iv) In reality, there will be resistive forces opposing the motion of the rubber ring. Explain how this fact will affect your answer to (iii). [1]

(b) Show that the rubber ring will remain in contact with the slide along the arc *AB*. [3]

END OF PAPER

